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Self-avoiding walks with curvature energy on fractals

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Abstract. Self-avoiding walks with a curvature-dependent energy are studied with renormalization group methods on some fractal lattices. Fixed points corresponding to universal and non-universal behaviours are generally present. However initial conditions of the renormalization group recursions can prevent non-universality. When universality holds the persistence length is found to diverge much faster than in the periodic lattice as the curvature energy increases.

The self-avoiding walk (SAW) problem is believed to catch all the essential features of real polymers in good solvent at least as far as their asymptotic behaviour is concerned [1, 2]. SAWs with extra interactions (besides the excluded volume) describe various situations. Attractions between different parts of a SAW have been introduced to describe the Θ-point tricritical behaviour and the collapsed phase (see [2] and reference therein). Closed SAWs in two dimensions with an unbalanced pressure have been studied as a model for two-dimensional vescicles [3].

Universality arguments make rather plausible that curvature energy on SAWs is irrelevant, the only effect of it being to modify the persistence length of the walk.

Let us consider for example SAWs on a d-dimensional (hyper)cubic lattice whose elementary steps joins nearest-neighbour lattice sites. Let k be the step fugacity and ϵ the (reduced) energy associated to each couple of consecutive steps at right angle (figure 1).

If $\epsilon \ll -1$, i.e. corners are very favourable, then SAWs become critical at the value $k_c(\epsilon) \sim e^{\epsilon}$, corresponding to having a turn associated at each step. It is easy to see that similar walks are nothing other than very fuzzy SAWs (persistence length ~ 1 in lattice units), and that a hypothetical renormalization group transformation would make them more 'smooth' at small scale.

On the other hand, if $\epsilon \gg 1$, corners become rather rare and the SAWs are made of rather long segments of length $l(\epsilon) \sim$ persistence length, which increases as a function of ϵ. This means that walks can be seen as new SAWs with elementary steps of length $l(\epsilon)$, the number of which is reduced by a factor $l(\epsilon)$.

Thus one expects that the average squared radius of gyration $\langle R^2 \rangle$ behaves like

$$\langle R^2 \rangle \sim l^2(\epsilon)(k_c(\epsilon) - k)^{-2\nu} \tag{1}$$

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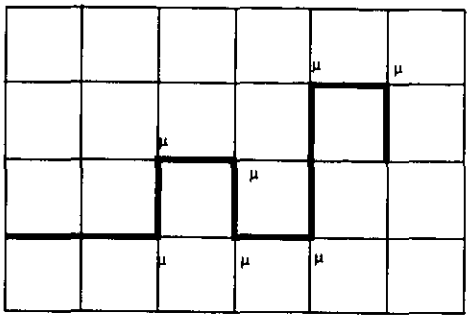

Figure 1. Example of the weight of the curvature; the weight here is $k^{10}e^{-7\epsilon}$.

as $k \to k_c(\epsilon)^-$ with the correlation length exponent ν independent of ϵ. Using the renormalization group approach of [4] we find (see also below) that $l(\epsilon) \sim e^{\epsilon}$ as $\epsilon \to \infty$ for all $d > 1$.

It is not known what could be the combined effect of a curvature energy cost for a SAW in a disorder environment. In some instances the environment forces the SAW to bend, constraining in this way the persistence length. Thus a modification to the scaling law (1) might be expected, with ν a (not necessarily continuous) function of ϵ.

As a first step in order to understand the role of the 'disorder', on a 'rigid' SAW it is useful to derive some exact results studying the problem on deterministic fractals. SAWs on deterministic and statistical fractals have been studied in [5] and [4] respectively.

We will present three typical examples where the correlation length exponent ν and the entropic exponent γ are: both non-universal; one universal and the other non-universal; and finally both universal. In this last case we will show that the persistence length grows like $e^{(e^{\epsilon})}$ as ϵ increases. We will also argue that non-universal asymptotic behaviour, even if always latent, might be unobservable due to initial conditions of renormalization group (RG) recursion equations that prevent the RG trajectories approaching new fixed points.

Renormalization group (RG) on deterministic fractal lattices was explained by Dhar [5]. A set of generating functions† for SAWs is introduced at the Nth iteration of the process which defines the construction of the fractal lattice. Recursion equations are then written for this set of generating functions which allow them to be calculated in terms of the same set of the previous iteration. The 'physical' parameters of the problem, k and $\mu \equiv e^{-\epsilon}$ in this case, enter in the recursion equations as initial conditions and/or as free parameters. The generating functions one introduces can be seen as new physical parameters in terms of which recursion equations can be written.

Let us illustrate the procedure in some detail for the simplest case of the branching Koch curve (BKC) [7], shown in figure 2.

Since each time we triple the linear size the number of elementary units increases by five, its fractal dimension is $\bar{d} = \log 5 / \log 3$.

Let $a_N(|w|)$ be the number of SAWs joining the two ends i, j as shown in figure 3, at the stage N, after $|w|$ steps; also let $b_N(|w|)$ be the number of SAWs starting from one end and ending on an arbitrary site, and $c_N(|w|)$ the number of coupled walks

† Which kinds of generating functions belong to the set depends not only on the type of quantities one likes to calculate, but also on the requirement that the recursion equations map the set into itself.

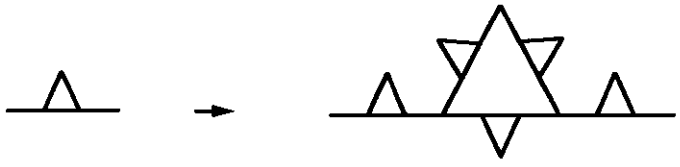

Figure 2. First steps in the construction of the branching Koch curve.

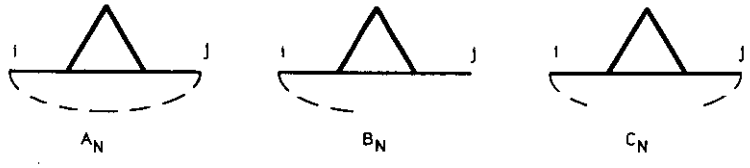

Figure 3. Generating functions A_N, B_N, C_N for the branching Koch curve as defined in the text.

of total length $|w|$ (figure 3).

Each walk is weighted as $k^{|w|}\mu^{C(w)}$. Here $\mu = e^{-\epsilon}$, and $C(w)$ is the number of corners present in the walk. It is useful to define the generating functions:

$$A_N(k) = \sum_{\substack{w: i \to j \\ \partial w = (i,j)}} k^{|w|}\mu^{C(w)} = \sum_{|w|=0}^{+\infty} a_N(|w|)k^{|w|}\mu^{C(w)} \tag{2}$$

$$B_N(k) = \sum_{w: i \in \partial w} k^{|w|}\mu^{C(w)} = \sum_{|w|=0}^{+\infty} b_N(|w|)k^{|w|}\mu^{C(w)} \tag{3}$$

and a similar expression for $C_N(k)$, the generating function for $c_N(|w|)$.

Recursion equations for A and B are easily constructed: for example two types of graphs contribute to A_{N+1} corresponding to go through the shortest and the longest coarse-grained paths. Taking into account the contributions to the extra statistical weight coming from the bends occurring at the nodes where the Nth lattices meet in the construction of the $(N+1)$th level iteration lattice, we get

$$A_{N+1} = A_N^3 + \mu^3 A_N^4. \tag{4}$$

The contributions to B_{N+1} and C_{N+1} are slightly more complicated, but easily obtainable:

$$B_{N+1} = B_N[1 + (1+\mu)A_N + (1+\mu+\mu^2)A_N^2 + (\mu^2+2\mu^3)A_N^3] - (\mu^2+\mu^3)A_N^4 \tag{5}$$

$$\begin{aligned} C_{N+1} = {} & C_N[3A_N^2 + 4\mu^3 A_N^3] + B_N^2 + 3\mu^2 A_N^2(B_N^2 - A_N^2) + 2\mu A_N B_N^2 \\ & + 2A_N(B_N^2 - A_N^2) + 2\mu A_N^2 B_N(B_N - A_N) \\ & + 2\mu A_N^2 B_N^2 + 4\mu^2 A_N^3(B_N - A_N)^2 \end{aligned} \tag{6}$$

where the negative contributions in (5) and (6) come from double counting of some graphs.

The initial conditions are: $A_0 = k, B_0 = k, C_0 = 0$. Since the recursion for A_N is not coupled, it can be solved separately. The fixed point $A^*(\mu)$ and the thermal eigenvalue $\lambda_T(\mu)$ are found from

$$A^{*2}(\mu) + \mu^3 A^{*3}(\mu) = 1 \tag{7}$$

and

$$\lambda_T(\mu) = \left.\frac{dA_{N+1}}{dA_N}\right|_{A=A^*} = 3 + \mu^3 A^{*3}(\mu). \tag{8}$$

The fact that the fixed point $A^*(\mu)$ and the thermal eigenvalue $\lambda_T(\mu)$ are explicitly dependent on the curvature parameter μ, has as a consequence that all the critical exponents will depend on μ as well. The end-to-end distance exponent defined in (1), can be read off from (8) to be

$$\nu(\mu) = \frac{\log 3}{\log[3 + \mu^3 A^{*3}(\mu)]}. \tag{9}$$

The case $\mu = 0$ corresponds to the infinitely rigid SAW and thus $\nu(0) = 1$ as one expects while $\nu(\mu = 1) = 0.891$ is the result of [7].

In this case, the critical value for the fugacity $k_C(\mu)$ coincides with $A^*(\mu)$. If we then let $\mu \gg 1, k_C(\mu) \ll 1$, such that $\mu k_C(\mu) = 1$, then we find from (5) that the SAW fractal dimension $1/\nu(\mu) = \log 4/\log 3$, coincides with the Koch fractal dimension and represents the most collapsed SAW on the BKC.

Let $f_N(|w|)$ be the the average number of closed SAWs per site (i.e. $\partial w = \emptyset$) of $|w|$ steps, at the stage N. Consider then its thermodynamic limit $f(|w|) = \lim_{N\to\infty} f_N(|w|)/5^N$. Then we can consider the generating function (free energy)

$$F(k) = \sum_{w:\partial w=\emptyset} k^{|w|}\mu^{c(w)} = \sum_{|w|=1}^{+\infty} f(|w|)k^{|w|}. \tag{10}$$

This is indeed the free energy for the SAW as it follows from the $n \to 0$ limit of the n-vector model [8]. Assume that $f(|w|)$ behaves like

$$f(|w|) \overset{|w|\gg 1}{\sim} k_C^{-|w|}|w|^{\alpha-3} \tag{11}$$

then [5] the free energy $F(k)$ has the behaviour

$$F(k) \overset{k\to k_C^-}{\sim} (k_C - k)^{2-\alpha} \tag{12}$$

where less singular terms have been omitted.

It is easy to see, using the definitions of $A_N(k)$, that

$$F(k) = \sum_{N=1}^{\infty} \mu^3 \frac{A_{N-1}^3}{5^N}. \tag{13}$$

From (13) it follows that

$$F(k) = \mu^3 A_0^3 + \sum_{N=2}^{\infty} \mu^3 \frac{A_{N-1}^3}{5^N} = \mu^3 A_0^3 + F(k^3 + \mu^3 k^4)/5$$

where we have used the fact that A_{N-1} with initial condition $A_0 = k$ is the same as A_{N-2} with initial condition $A_1 = k^3 + \mu^3 k^4$ (see (4)).

Thus the free energy exponent as defined in (12) becomes:

$$\alpha(\mu) = 2 - \frac{\log 5}{\log[3 + \mu^3 A^{*3}(\mu)]}. \tag{14}$$

It is worth noticing that the hyperscaling relation $\bar{d}\nu = 2 - \alpha$ is verified $\forall \mu \in [0, +\infty)$.

The method of calculation of the entropy exponent γ takes the same line. Assume that the average number of distinct SAWs of $|w|$ steps at the stage N is $c_N(|w|)$. Then define $c(|w|) = \lim_{N\to\infty} c_N(|w|)/5^N$ the average number of $|w|$-step SAW per site. Since we expect

$$c(|w|) \overset{|w|\gg 1}{\sim} k_C^{-|w|} |w|^{\gamma - 1} \tag{15}$$

then the susceptibility $\chi(k) = \sum_{|w|=0}^{+\infty} c(|w|) k^{|w|}$ for $k \sim k_c$ is

$$\chi(k) \overset{k\to k_C^-}{\sim} (k_C - k)^{-\gamma}. \tag{16}$$

Using the definitions of A, B and C it is easy to obtain [5]

$$\begin{aligned}\chi(k) = \sum_{N=1}^{+\infty} \frac{1}{5^N} [&(2 + 5\mu) B_{N-1}^2 + (1 + 2\mu + 5\mu^2) A_{N-1} B_{N-1}^2 \\ &+ (2\mu^2 + 3\mu^3) A_{N-1}^2 B_{N-1}^2 + 3\mu^3 C_{N-1} A_{N-1}^2 - 2(\mu^2 + \mu^3) B_{N-1} A_{N-1}^2 \\ &- 3\mu^2 A_{N-1}^3].\end{aligned} \tag{17}$$

Linearizing (4) in the vicinity of the fixed point one has $\delta_N = \lambda_N \delta_{N-1}$, where $\delta_N = A^* - A_N > 0$. This will hold approximately for $N \in (N_0, \bar{N})$, where N_0, for $\delta_0 \ll 1$, is constant and $\bar{N} \to \infty$ as $\delta_0 \to 0$; $\bar{N}$ can be defined such that $\delta_{\bar{N}} = \lambda_T^{\bar{N}} \delta_0 = \delta \ll 1$, i.e.

$$\bar{N} = \frac{\log(\delta/\delta_0)}{\log(\lambda_T(\mu))}. \tag{18}$$

Then for $N_0 < N < \bar{N}$ we have $A_N \sim A^*(\mu)$ and $B_N \sim [D(\mu)]^N$ where

$$D(\mu) = 1 + (1 + \mu) A^*(\mu) + (1 + \mu + \mu^2) A^{*2}(\mu) + (\mu^2 + 2\mu^3) A^{*3}(\mu) \tag{19}$$

while for $N > \bar{N}$, $A_N \sim 0$ and $B_N \sim B_{\bar{N}}$ due to the condition $k \to k_C^-(\mu)$. Under this condition we can approximate the sum in (17) with its maximum term, thus leading to

$$\chi(k) \sim \left(\frac{D^2(\mu)}{5}\right)^{\bar{N}} \sim \delta_0^{-\gamma(\mu)} \sim (k_C(\mu) - k)^{-\gamma(\mu)} \tag{20}$$

yielding the susceptibility exponent

$$\gamma(\mu) = \frac{\log(D^2(\mu)/5)}{\log(\lambda_T(\mu))}. \tag{21}$$

In a similar way, for the γ_1 exponent associated with the divergence of the susceptibility defined considering only walks leaving one edge of the unit at the stage N, we obtain

$$\chi_1(k) = \lim_{N\to\infty} B_N. \tag{22}$$

The same considerations given above for $\gamma(\mu)$ apply, and

$$\gamma_1(\mu) = \frac{\log D(\mu)}{\log \lambda_T(\mu)}. \tag{23}$$

A summary of all the exponents are given in table 1, while the behaviour of the exponents $\alpha(\mu), \nu(\mu), \gamma(\mu), \gamma_1(\mu)$ as function of μ is shown in figure 4(*a*) and (*b*).

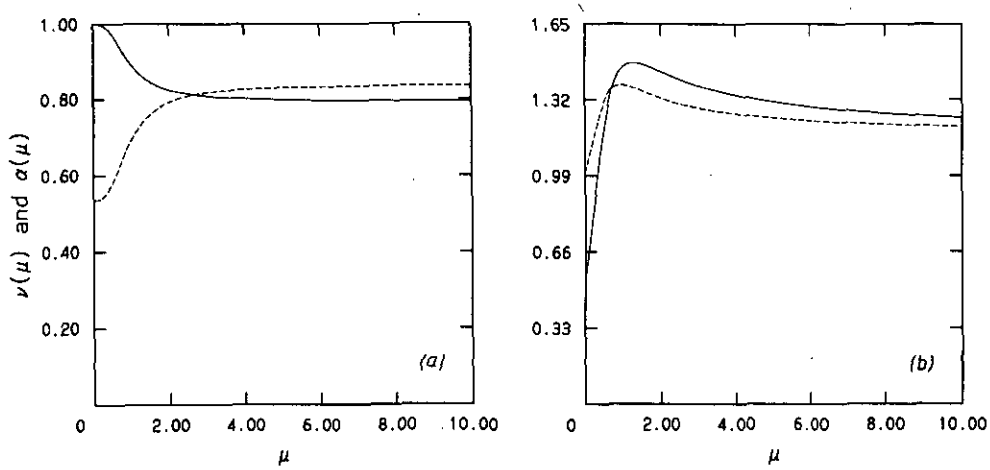

Figure 4. Behaviour of (*a*) $\alpha(\mu)$ (dotted curve), and $\nu(\mu)$ (full curve) and (*b*) $\gamma(\mu)$ (full curve), and $\gamma_1(\mu)$ (dotted curve) as a function of the curvature μ in the case of the branched Koch curve (BKC).

Table 1. Summary table for the various lattices studied in the text. The non-universal (μ-dependent) exponents are given in the text in equations (9), (14), (21) and (23) for the BKC, and in equations (26*a*)–(26*c*) for the HBA.

Model	ν	α	γ	γ_1
BKC	$\nu(\mu)$	$\alpha(\mu)$	$\gamma(\mu)$	$\gamma_1(\mu)$
3-simplex	0.798	0.734...	1.375...	1.320...
HBA	1	0.415...	$\gamma(\mu)$	$\gamma_1(\mu)$

It is worth noticing that a scaling relation relating the surface and the bulk exponents holds. Indeed from the above equations we have

$$\gamma(\mu) = 2\gamma_1(\mu) + \alpha(\mu) - 2 \tag{24}$$

for all μ.

In this simple example non-universality comes from the μ-dependence of the recursion equation (4). In the next example recursion equations can be introduced without μ-dependence. Nevertheless entropic exponents will be non-universal.

Let us consider a fractal bearing only reflection symmetry introduced by Havlin and Ben-Avraham (HBA) [9], shown in figure 5(*a*); its fractal dimension is $\bar{d} = \log 3/\log 2$. In this case we need 10 different generating functions $\{A_N, B_N^{(1)}, B_N^{(2)}, C_N, D_N, E_N^{(1)}, E_N^{(2)}, F_N, G_N, H_N\}$ whose definitions are shown in figure 6. The initial conditions are $\{A_0 = \mu k^2, B_0^{(1)} = k, B_0^{(2)} = 0, C_0 = k(1+\mu k) D_0 = k(1+\mu), E_0^{(1)} = E_0^{(2)} = F_0 = G_0 = H_0 = 0\}$. The recursions for $A_N, B_N^{(1)}, B_N^{(2)}$ are coupled together but decoupled from $C_N, \ldots, H_N$. A convenient redefinition of the variables is $\{X_N = B_N^{(1)} + \mu B_N^{(2)}, Y_N = B_N^{(2)} + \mu B_N^{(1)}, Z_N = \mu A_N\}$.

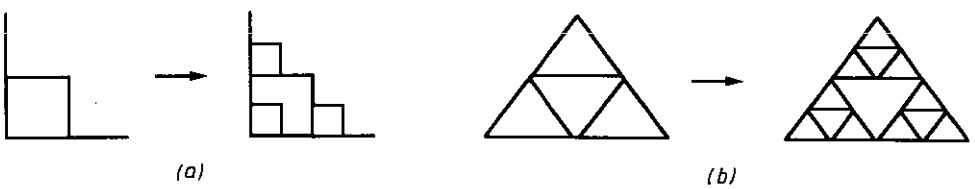

Figure 5. First steps in the construction of (*a*) the Havlin–Ben-Avraham model and (*b*) the 3-simplex model.

In these variables the recursions assume the simple form:

$$X_{N+1} = X_N^2 + Y_N^2 Z_N \tag{25a}$$

$$Y_{N+1} = X_N Y_N + X_N Y_N Z_N \tag{25b}$$

$$Z_{N+1} = X_N^2 Z_N + Z_N^2 \tag{25c}$$

with initial condition $\{X_0 = k, Y_0 = \mu k, Z_0 = \mu^2 k^2\}$. The only non-trivial fixed points are $P = (X^* = 0, Y^* = 0, Z^* = 1)$, $S = (X^* = \Delta, Y^* = \Delta, Z^* = \Delta)$ where $\Delta = (\sqrt{5} - 1)/2$, and a line of fixed points $L = (X^* = 1, Y^*, Z^* = 0)$ where here Y^* is arbitrary. The line of fixed points L has two marginal direction $\lambda_{1,2} = 1$ and one repulsive direction $\lambda_3 = 2$ independent of the free parameter Y^*! Although S does have an attractive eigendirection corresponding to the eigenvalues $\lambda_1 = 2\Delta - 1 = 0.236\ldots$ (the other two being $\lambda_2 = 2\Delta = 1.23\ldots, \lambda_3 = 2 + \Delta^2 = 2.38\ldots$), it does not intersect the initial condition surface, and thus is never involved in the scaling regime. At the fixed point P, $\lambda_{1,2} = 0$ and $\lambda_3 = 2$.

At least for $\mu \lesssim 2$ we verified that iterating the recursions equations (25*a*)–(25*c*), there exists a critical value of the fugacity $k_C(\mu)$ leading (X_N, Y_N, Z_N) to one of the points L as $N \to \infty$. Therefore one gets $\nu = 1$ since $\lambda_3 = 2$ on that line. For larger values of μ it is not completely clear from the numerical investigation of the

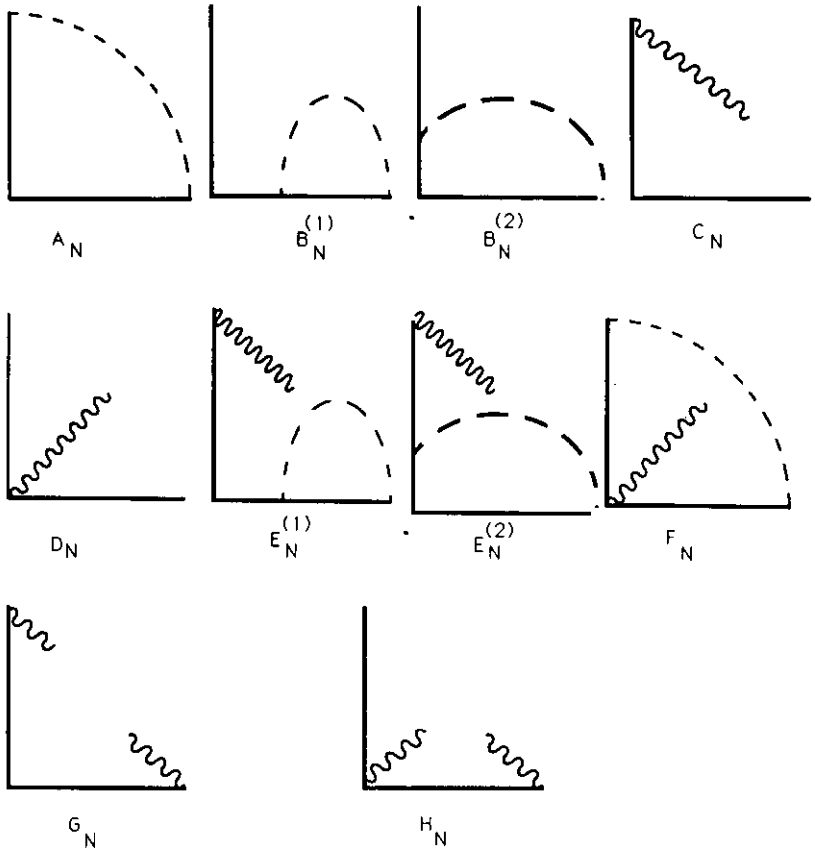

Figure 6. Generating functions for the Havlin–Ben-Avraham fractal. In the graphs $B_N^{(i)}$ and $E_N^{(i)}$ $(i = 1, 2)$, the dashed line joins the corner to the extremum site on the right. In the case $i = 1$ and $i = 2$ paths arrive at the corner horizontally and vertically respectively.

recursion equations, if criticality is regulated by the fixed point P or by one of the Ls since the RG trajectory, as one approaches $k_C(\mu)$, first goes through a neighbourhood of P and then through one of the Ls. We have also calculated perturbatively the surface of attraction of the fixed point P and of the set $\{L\}$. Within the perturbative scheme we have evidence that the surface of the initial conditions does not intersect the domain of attraction of P thus leading to a critical behaviour regulated by the fixed points in the set $\{L\}$.

This model, without curvature, has been investigated by Rammal *et al* [7] who nevertheless considered the ν exponent only. Their case is a particular case of the present one when $\mu = 1$, that is $X_N = Y_N, \forall N \geqslant 0$. Although we introduced one more degree of freedom, thus enlarging the parameter space with an attractive eigendirection, we found the same result. In this respect then, the effect of the curvature is irrelevant.

Using the same method as before, we also calculated the exponents α, γ, γ_1: while α turns out to be independent of curvature thus leaving the hyperscaling relation

unchanged, the susceptibility exponents γ and γ_1 do not!† Their values are

$$\gamma(\mu) = \frac{\log[2(2 + Y^*(\mu))/3]}{\log \lambda_T} \tag{26a}$$

$$\gamma_1(\mu) = \frac{\log 2}{\log \lambda_T} = 1 \tag{26b}$$

$$\gamma_1'(\mu) = \frac{\log[2 + Y^*(\mu)]}{\log \lambda_T}. \tag{26c}$$

The values γ_1 and γ_1' correspond to the susceptibility of the extreme and the corner respectively.

A relation similar to (24) holds true, namely

$$\gamma(\mu) = \gamma_1 + \gamma_1'(\mu) + \alpha - 2 \tag{27}$$

Thus in this second example, in contrast with the previous one, only part of the exponents are non-universal.

The next case, i.e. the 3-simplex [5] (figure 5(*b*)), is more similar to the regular lattice problem where universality holds, but is nevertheless instructive since it allows definition and calculation, within the RG framework, of the persistence length in the small μ limit. This model has the three-fold symmetry characteristic also of the Sierpinski gasket family, and it is believed to belong to the same class of universality. Notice that, although it has the same fractal dimension $\bar{d} = \log 3 / \log 2$ of the HBA lattice, it does not belong to the same class of universality [7]. For the calculation of the exponent ν, the generating functions $\{A_N^{(1,1)}, A_N^{(1,2)}, A_N^{(2,1)}, A_N^{(2,2)}\}$ we need are shown in figure 7.

The redefinition

$$X_N = A_N^{(1,1)} + \mu A_N^{(2,1)} \tag{28a}$$

$$Y_N = A_N^{(2,1)} + \mu A_N^{(1,1)} \tag{28b}$$

$$V_N = A_N^{(1,2)} + \mu A_N^{(2,2)} \tag{28c}$$

$$Z_N = A_N^{(2,2)} + \mu A_N^{(1,2)} \tag{28d}$$

along with the initial conditions $\{X_0 = k^2 + \mu^3 k^3, Y_0 = \mu^2 k^3 + \mu k^2, V_0 = \mu^2 k^3 + \mu k^2, Z_0 = \mu k^3 + \mu^2 k^2\}$, allow the reduction of the problem to consider the 3D map $\{X_N, Y_N, Z_N\}$ only, being $V_N = Y_N \; \forall N \geqslant 0$. The recursions are

$$X_{N+1} = X_N^2 + Y_N^2 Z_N \tag{29a}$$

$$Y_{N+1} = X_N Y_N + X_N Y_N Z_N \tag{29b}$$

$$Z_{N+1} = Y_N^2 + X_N^2 Z_N. \tag{29c}$$

† The calculation of the γs exponents would require the introduction of two generating functions of type A: one involving paths going through the corner of figure 6 and the other the remaining paths. This complication however does not change the result for the exponents.

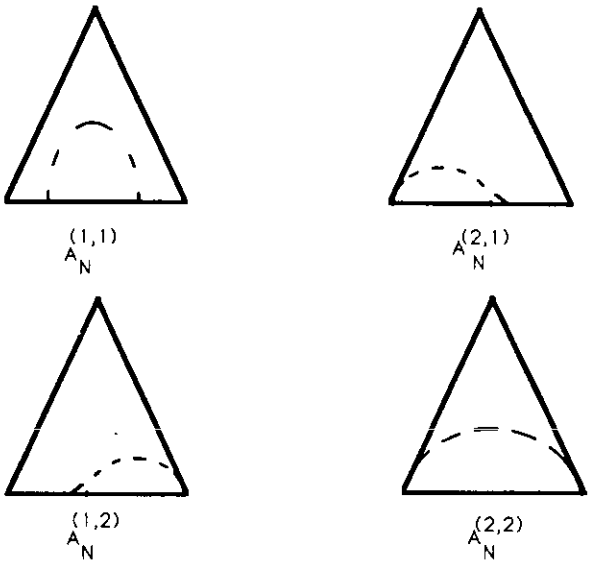

Figure 7. Generating functions for the 3-simplex fractal. Only the graphs necessary for the calculation of ν are reported. Paths join the left to the right corner. $A_N^{(i,j)}$ means that the left corner is reached with a horizontal (vertical) step for $i = 1$ (2). The index j refers in the same way to the right corner.

Notice that only the recursion for Z is different from the HBA case, beside the initial conditions. Now we find again a symmetric fixed point $S = (X^* = \Delta, Y^* = \Delta, Z^* = \Delta)$ with $\Delta = (\sqrt{5} - 1)/2$ with eigenvalues $(\lambda_1 = 0, \lambda_2 = 2\Delta - 1, \lambda_3 = 2 + \Delta^2 = 2.38\ldots)$, and a line of fixed points $L = (X^* = 1, Y^* = 0, Z^*)$, with eigenvalues $(\lambda_1 = 2, \lambda_2 = 1, \lambda_3 = 1 + Z^*)$ and Z^* is arbitrary. Due to the initial condition surface, the flow is toward the symmetric fixed point S, yielding $\lambda_T = 2 + \Delta^2$ and thus $\nu = \log 3/\log(2 + \Delta^2) = 0.7986\ldots$. This model, without curvature, was studied by Dhar [5], whose recursions can be regained from (29*a*)–(29*c*) and the initial conditions, when $\mu = 1$, (i.e. $X_N = Y_N = Z_N, \forall N \geqslant 0$). Therefore, again, we find a result independent of the curvature, which coincides with the $\mu = 1$ solution. The calculation of the entropic exponents of γ takes the same line as in [5]. In contrast with the previous case we find now that the γ and γ_1 are universal. Thus the line of fixed points, L, in this case does not play any role since the initial condition surface does not intersect its domain of attraction.

It is interesting to see for this particular case, where universality holds, what is the effect of the inhomogeneous environment in the limit of very rigid chain, i.e. $\mu \to 0$. Specifically we wish to evaluate the singular behaviour of the persistence length $l(\mu)$ as defined in (1). When μ is very small, the starting point (X_0, Y_0, Z_0) is near the fixed point $(1,0,0)$ of the infinitely rigid chain. The smaller μ is, the more the RG trajectory stays around that fixed point before approaching the symmetric fixed point S. Since we are interested in the critical behaviour, one has to start on the critical surface, which in this case is the domain of attraction of S and contains points arbitrarily closed to $(1,0,0)$, and then determines how many recursions are needed to leave a small neighbourhood of $(1,0,0)$. Let us call $n_0(\mu)$ this number which will also be of the order of the number of recursions needed to approach S. The

persistence length is then proportional to $b^{n_0(\mu)}$ where b is the rescaling factor, equal to 2 in the present case. To leading order the recursions near $(1,0,0)$ and on the critical surface are

$$Y_{N+1} = Y_N(1 + Z_N) \tag{30a}$$

$$Z_{N+1} = Z_N + Y_N^2 \tag{30b}$$

with initial conditions $Z_0 \sim Y_0 \sim \mu \sim 0$. One finds easily that $n_0(\mu) \sim \mu^{-1}$ and thus

$$\log l(\mu) \sim n_0(\mu) \sim \mu^{-1} \tag{31}$$

Using the exact result of [4] concerning the stiff-to-isotropic crossover exponent, the above analysis lead to

$$l(\mu) \sim \mu^{-1} \tag{32}$$

for d-dimensional hypercubic lattice. Thus (31) suggests that, when universality holds, the persistence length for very rigid SAWs on fractals is much larger than the one on the regular lattices.

In summary, we have presented an analytical investigation of the effect of the curvature on a SAW which is taking place on a fractal structure. Although we found that in some instances a non-universal behaviour may occur, we showed that in the framework of the deterministic fractals, there is no general rule and a non-universal behaviour may or may not occur. It is however extremely important that non-universal behaviour is always present. Indeed in all three typical examples we have presented, when universal behaviour occurs, it is only due to the particular initial conditions, while non-universal behaviour is always latent. Furthermore if the end-to-end distance exponent ν is universal, then our calculations indicate that the persistence length is much larger than the one in the regular lattice when curvature energy does not favour corners. Further work will be necessary in the future, to see the effect of curvature energy on SAWs on statistical fractals.

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References

[1] deGennes P G 1979 *Scaling Concepts in Polymer Physics* (Ithaca, NY: Cornell University Press)

[2] des Closeaux J and Jannik G 1978 *Les Polymèeres en Solution: leur Modelization et leur Structure* (Paris: Editions de Physique)
[3] Lieber S, Singh R and Fisher M 1987 *Phys. Rev. Lett.* **59** 1989
Banavar J, Maritan A and Stella A L 1991 *Phys. Rev.* A **43** 5752
[4] Lee S B and Nakanishi H 1987 *J. Phys. A: Math. Gen.* **20** LA57
[5] Dhar D 1978 *J. Math. Phys.* **19** 5
Klein D J and Seitz W A 1984 *J. Physique* **45** L241
[6] Lee S B and Nakanishi H 1988 *Phys. Rev. Lett.* **61** 2022
[7] Rammal R, Toulouse G and Vannimenus J 1984 *J. Physique* **47** 389
[8] Hilhorst H J 1977 *Phys. Rev.* B **16** 1253 and references therein
[9] Havlin S and Ben-Avraham 1983 *J. Phys. A: Math. Gen.* **16** L837
[10] Tuthill G F and Schwalm W A *Preprint*
[11] Sampson B 1991 *MS Thesis* University of North Dakota